Evaluation of the Performance
of Randomized FFD Control Grids

Master Thesis

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by

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Contents

How to read this Thesis

As a guide through the nomenclature used in the formulas we prepend this chapter.

Unless otherwise noted the following holds:

- lowercase letters x,y,z

  refer to real variables and represent a point in 3D–Space.

- lowercase letters u,v,w

  refer to real variables between 0 and 1 used as coefficients in a 3D B–Spline grid.

- other lowercase letters

  refer to other scalar (real) variables.

- lowercase **bold** letters (e.g. $\mathbf{x},\mathbf{y}$)

  refer to 3D coordinates

- uppercase **BOLD** letters (e.g. $\mathbf{D}, \mathbf{M}$)

  refer to Matrices

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1 | **Introduction**

Many modern industrial design processes require advanced optimization methods do to the increased complexity. These designs have to adhere to more and more degrees of freedom as methods refine and/or other methods are used. Examples for this are physical domains like aerodynamic (i.e. drag), fluid dynamics (i.e. throughput of liquid) — where the complexity increases with the temporal and spatial resolution of the simulation — or known hard algorithmic problems in informatics (i.e. layouting of circuit boards or stacking of 3D–objects). Moreover these are typically not static environments but requirements shift over time or from case to case.

Evolutional algorithms cope especially well with these problem domains while addressing all the issues at hand[1]. One of the main concerns in these algorithms is the formulation of the problems in terms of a genome and a fitness function. While one can typically use an arbitrary cost–function for the fitness–functions (i.e. amount of drag, amount of space, etc.), the translation of the problem–domain into a simple parametric representation can be challenging.

The quality of such a representation in biological evolution is called *evolvability*[2] and is at the core of this thesis. However, there is no consensus on how *evolvability* is defined and the meaning varies from context to context[3].

As we transfer the results of Richter et al.[4] from using Radial Basis Function (RBF) as a representation to manipulate a geometric mesh to the use of Freeform–Deformation (FFD) we will use the same definition for evolvability the original author used, namely *regularity*, *variability*, and *improvement potential*. We introduce these term in detail in Chapter 2.4.

In the original publication the author used random sampled points weighted with Radial Basis Function (RBF) to deform the mesh and showed that the mentioned criteria of *regularity*, *variability*, and *improvement potential* correlate with the quality and potential of such optimization.

We will replicate the same setup on the same meshes but use Freeform–Deformation (FFD) instead of Radial Basis Function (RBF) to create a local deformation near the control points and evaluate if the evolution–criteria still work as a predictor given the different deformation scheme, as suspected in [4].

1.1   Outline of this thesis

First we introduce different topics in isolation in Chapter 2. We take an abstract look at the definition of FFD for a one–dimensional line (in 2.1) and discuss why this is a sensible deformation function (in 2.1.1). Then we establish some background–knowledge of evolutional algorithms (in 2.2) and why this is useful in our domain (in 2.3). In a third step we take a look at the definition of the different evolvability criteria established in [4].

In Chapter 3 we take a look at our implementation of FFD and the adaptation for 3D–meshes.

Next, in Chapter 4, we describe the different scenarios we use to evaluate the different evolvability–criteria incorporating all aspects introduced in Chapter 2. Following that, we evaluate the results in Chapter 5 with further on discussion in Chapter 6.

2 | Background

2.1   What is Freeform–Deformation (FFD)?

First of all we have to establish how a FFD works and why this is a good tool for deforming
meshes in the first place. For simplicity we only summarize the 1D–case from [5] here and
go into the extension to the 3D case in chapter 3.2.

Given an arbitrary number of points p_i alongside a line, we map a scalar value $\tau_i \in [0,1[$
to each point with $\tau_i < \tau_{i+1} \forall i$. Given a degree of the target polynomial d we define the
curve $N_{i,d,\tau_i}(u)$ as follows:

$$N_{i,0,\tau}(u) = \begin{cases} 1, & u \in [\tau_i, \tau_{i+1}[\\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

and

$$N_{i,d,\tau}(u) = \frac{u - \tau_i}{\tau_{i+d}} N_{i,d-1,\tau}(u) + \frac{\tau_{i+d+1} - u}{\tau_{i+d+1} - \tau_{i+1}} N_{i+1,d-1,\tau}(u) \tag{2.2}$$

If we now multiply every p_i with the corresponding $N_{i,d,\tau_i}(u)$ we get the contribution of
each point p_i to the final curve–point parameterized only by $u \in [0,1[$. As can be seen from
(2.2) we only access points $[i..i + d]$ for any given i[1], which gives us, in combination with
choosing p_i and τ_i in order, only a local interference of $d + 1$ points.

We can even derive this equation straightforward for an arbitrary N[2]:

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[1] one more for each recursive step.

[2] *Warning:* in the case of $d = 1$ the recursion–formula yields a 0 denominator, but N is also 0. The right
  solution for this case is a derivative of 0

$$\frac{\partial}{\partial u} N_{i,d,r}(u) = \frac{d}{\tau_{i+d} - \tau_i} N_{i,d-1,\tau}(u) - \frac{d}{\tau_{i+d+1} - \tau_{i+1}} N_{i+1,d-1,\tau}(u)$$

For a B–Spline

$$s(u) = \sum_i N_{i,d,\tau_i}(u) p_i$$

these derivations yield $\frac{\partial^d}{\partial u} s(u) = 0$.

Another interesting property of these recursive polynomials is that they are continuous (given $d \geq 1$) as every p_i gets blended in linearly between τ_i and τ_{i+d} and out linearly between τ_{i+1} and τ_{i+d+1} as can bee seen from the two coefficients in every step of the recursion.

2.1.1 Why is FFD a good deformation function?

The usage of FFD as a tool for manipulating follows directly from the properties of the polynomials and the correspondence to the control points. Having only a few control points gives the user a nicer high–level–interface, as she only needs to move these points and the model follows in an intuitive manner. The deformation is smooth as the underlying polygon is smooth as well and affects as many vertices of the model as needed. Moreover the changes are always local so one risks not any change that a user cannot immediately see.

But there are also disadvantages of this approach. The user loses the ability to directly influence vertices and even seemingly simple tasks as creating a plateau can be difficult to achieve[6, chapter 3.2][7].

This disadvantages led to the formulation of Direct Manipulation Freeform–Deformation (DM–FFD)[6, chapter 3.3] in which the user directly interacts with the surface–mesh. All interactions will be applied proportionally to the control–points that make up the parametrization of the interaction–point itself yielding a smooth deformation of the surface *at* the surface without seemingly arbitrary scattered control–points. Moreover this increases the efficiency of an evolutionary optimization[8], which we will use later on.

But this approach also has downsides as can be seen in figure 2.1, as the tessellation of the invisible grid has a major impact on the deformation itself.
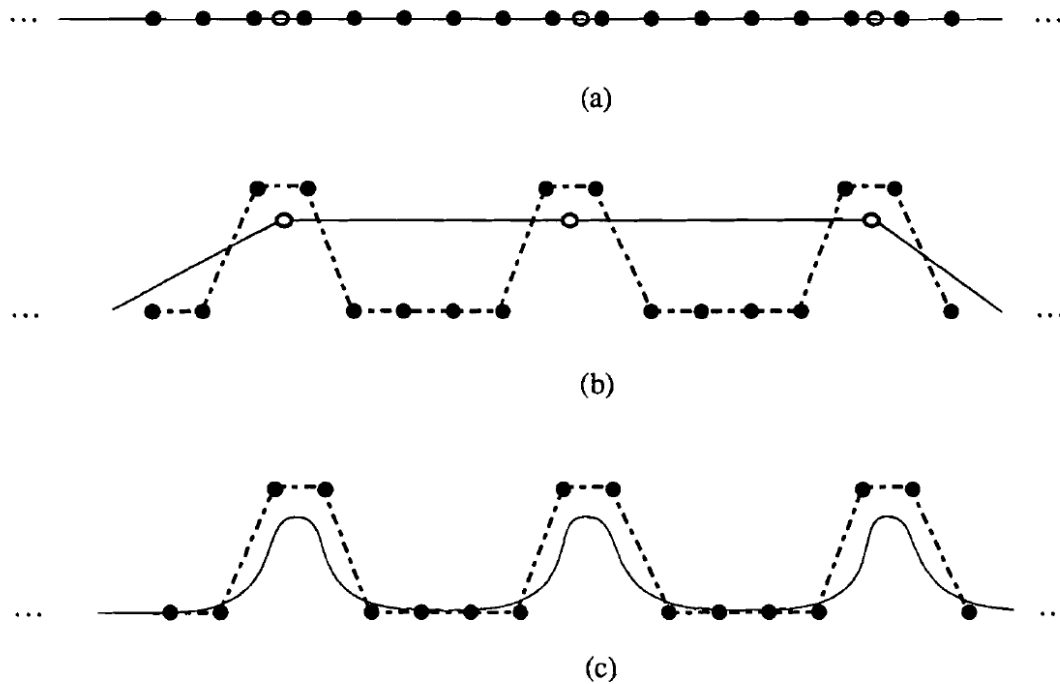
Figure 7. Aliasing because of too many control points relative to the number of object points. Open circles denote object points, filled circles denote control points. No control polygon is drawn. (a) Initial condition. (b) Resulting control-point location when the three object points are moved upward. (c) Possible aliasing if the resolution of the object is dramatically increased.

Figure 2.1: Figure 7 from [6].

All in all FFD and DM–FFD are still good ways to deform a high–polygon mesh albeit the downsides.

2.2 What is evolutional optimization?

Change: Write this section

2.3 Advantages of evolutional algorithms

Change: Needs citations

The main advantage of evolutional algorithms is the ability to find optima of general functions just with the help of a given error–function (or fitness–function in this domain).

This avoids the general pitfalls of gradient–based procedures, which often target the same error–function as an evolutional algorithm, but can get stuck in local optima.

This is mostly due to the fact that a gradient–based procedure has only one point of observation from where it evaluates the next steps, whereas an evolutional strategy starts with a population of guessed solutions. Because an evolutional strategy modifies the solution randomly, keeps the best solutions and purges the worst, it can also target multiple different hypothesis at the same time where the local optima die out in the face of other, better candidates.

If an analytic best solution exists (i.e. because the error–function is convex) an evolutional algorithm is not the right choice. Although both converge to the same solution, the analytic one is usually faster. But in reality many problems have no analytic solution, because the problem is not convex. Here evolutional optimization has one more advantage as you get bad solutions fast, which refine over time.

2.4 Criteria for the evolvability of linear deformations

2.4.1 Variability

In [4] *variability* is defined as

$$V(\mathbf{U}) := \frac{\text{rank}(\mathbf{U})}{n},$$

whereby $\mathbf{U}$ is the $m \times n$ deformation–Matrix used to map the m control points onto the n vertices.

Given $n = m$, an identical number of control–points and vertices, this quotient will be $= 1$ if all control points are independent of each other and the solution is to trivially move every control–point onto a target–point.

In praxis the value of $V(\mathbf{U})$ is typically $\ll 1$, because as there are only few control–points for many vertices, so $m \ll n$.

Additionally in our setup we connect neighbouring control–points in a grid so each control point is not independent, but typically depends on 4^d control–points for an d–dimensional control mesh.

2.4.2 Regularity

Regularity is defined[4] as

$$R(\mathbf{U}) := \frac{1}{\kappa(\mathbf{U})} = \frac{\sigma_{min}}{\sigma_{max}}$$

where σ_{min} and σ_{max} are the smallest and greatest right singular value of the deformation–matrix $\mathbf{U}$.

As we deform the given Object only based on the parameters as $\mathbf{p} \mapsto f(\mathbf{x} + \mathbf{U}\mathbf{p})$ this makes sure that $\|\mathbf{U}\mathbf{p}\| \propto \|\mathbf{p}\|$ when $\kappa(\mathbf{U}) \approx 1$. The inversion of $\kappa(\mathbf{U})$ is only performed to map the criterion–range to $[0..1]$, whereas 1 is the optimal value and 0 is the worst value.

This criterion should be characteristic for numeric stability on the on hand[9, chapter 2.7] and for convergence speed of evolutional algorithms on the other hand[4] as it is tied to the notion of locality[10, 11].

2.4.3 Improvement Potential

In contrast to the general nature of *variability* and *regularity*, which are agnostic of the fitness–function at hand the third criterion should reflect a notion of potential.

As during optimization some kind of gradient g is available to suggest a direction worth pursuing we use this to guess how much change can be achieved in the given direction.

The definition for an *improvement potential P* is[4]:

$$P(\mathbf{U}) := 1 - \|(\mathbf{1} - \mathbf{U}\mathbf{U}^{+})(G)\|_F^2$$

given some approximate $n \times d$ fitness–gradient $\mathbf{G}$, normalized to $\|\mathbf{G}\|_F = 1$, whereby $\|\cdot\|_F$ denotes the Frobenius–Norm.

3 | Implementation of Freeform–Deformation (FFD)

The general formulation of B–Splines has two free parameters d and τ which must be chosen beforehand.

As we usually work with regular grids in our FFD we define τ statically as $\tau_i = {}^i\!/n$ whereby n is the number of control–points in that direction.

d defines the *degree* of the B–Spline–Function (the number of times this function is differentiable) and for our purposes we fix d to 3, but give the formulas for the general case so it can be adapted quite freely.

3.1   Adaption of FFD

As we have established in Chapter 2.1 we can define an FFD–displacement as

$$\Delta_x(u) = \sum_i N_{i,d,\tau_i}(u)\Delta_x c_i \tag{3.1}$$

Note that we only sum up the Δ–displacements in the control points c_i to get the change in position of the point we are interested in.

In this way every deformed vertex is defined by

$$\text{Deform}(v_x) = v_x + \Delta_x(u)$$

with $u \in [0..1[$ being the variable that connects the high–detailed vertex–mesh to the low–detailed control–grid. To actually calculate the new position of the vertex we first have to

calculate the u–value for each vertex. This is achieved by finding out the parametrization of v in terms of c_i

$$v_x \stackrel{!}{=} \sum_i N_{i,d,\tau_i}(u)c_i$$

so we can minimize the error between those two:

$$\operatorname*{argmin}_u Err(u,v_x) = \operatorname*{argmin}_u 2 \cdot \left\| v_x - \sum_i N_{i,d,\tau_i}(u)c_i \right\|_2^2$$

As this error–term is quadratic we just derive by u yielding

$$\frac{\partial}{\partial u} \quad v_x - \sum_i N_{i,d,\tau_i}(u)c_i$$
$$= \quad -\sum_i \left(\frac{d}{\tau_{i+d}-\tau_i} N_{i,d-1,\tau}(u) - \frac{d}{\tau_{i+d+1}-\tau_{i+1}} N_{i+1,d-1,\tau}(u) \right) c_i$$

and do a gradient–descend to approximate the value of u up to an ε of 0.0001.

For this we use the Gauss–Newton algorithm[12] as the solution to this problem may not be deterministic, because we usually have way more vertices than control points ($\#v \gg \#c$).

3.2 Adaption of FFD for a 3D–Mesh

This is a straightforward extension of the 1D–method presented in the last chapter. But this time things get a bit more complicated. As we have a 3–dimensional grid we may have a different amount of control–points in each direction.

Given n,m,o control points in x,y,z–direction each Point on the curve is defined by

$$V(u,v,w) = \sum_i \sum_j \sum_k N_{i,d,\tau_i}(u) N_{j,d,\tau_j}(v) N_{k,d,\tau_k}(w) \cdot C_{ijk}.$$

In this case we have three different B–Splines (one for each dimension) and also 3 variables u,v,w for each vertex we want to approximate.

Given a target vertex $\mathbf{p}^*$ and an initial guess $\mathbf{p} = V(u,v,w)$ we define the error–function for the gradient–descent as:

$$Err(u,v,w,\mathbf{p}^*) = \mathbf{p}^* - V(u,v,w)$$

And the partial version for just one direction as

$$Err_x(u,v,w,\mathbf{p}^*) = p_x^* - \sum_i \sum_j \sum_k N_{i,d,\tau_i}(u) N_{j,d,\tau_j}(v) N_{k,d,\tau_k}(w) \cdot c_{ijk_x}$$

To solve this we derive partially, like before:

$$\frac{\partial Err_x}{\partial u} \quad p_x^* - \sum_i \sum_j \sum_k N_{i,d,\tau_i}(u) N_{j,d,\tau_j}(v) N_{k,d,\tau_k}(w) \cdot c_{ijk_x}$$
$$= \quad -\sum_i \sum_j \sum_k N'_{i,d,\tau_i}(u) N_{j,d,\tau_j}(v) N_{k,d,\tau_k}(w) \cdot c_{ijk_x}$$

The other partial derivatives follow the same pattern yielding the Jacobian:

$$J(Err(u,v,w)) = \begin{pmatrix} \frac{\partial Err_x}{\partial u} & \frac{\partial Err_x}{\partial v} & \frac{\partial Err_x}{\partial w} \\ \frac{\partial Err_y}{\partial u} & \frac{\partial Err_y}{\partial v} & \frac{\partial Err_y}{\partial w} \\ \frac{\partial Err_z}{\partial u} & \frac{\partial Err_z}{\partial v} & \frac{\partial Err_z}{\partial w} \end{pmatrix}$$

$$= \begin{pmatrix} -\sum_{i,j,k} N'_i(u)N_j(v)N_k(w) \cdot c_{ijk_x} & -\sum_{i,j,k} N_i(u)N'_j(v)N_k(w) \cdot c_{ijk_x} & -\sum_{i,j,k} N_i(u)N_j(v)N'_k(w) \cdot c_{ijk_x} \\ -\sum_{i,j,k} N'_i(u)N_j(v)N_k(w) \cdot c_{ijk_y} & -\sum_{i,j,k} N_i(u)N'_j(v)N_k(w) \cdot c_{ijk_y} & -\sum_{i,j,k} N_i(u)N_j(v)N'_k(w) \cdot c_{ijk_y} \\ -\sum_{i,j,k} N'_i(u)N_j(v)N_k(w) \cdot c_{ijk_z} & -\sum_{i,j,k} N_i(u)N'_j(v)N_k(w) \cdot c_{ijk_z} & -\sum_{i,j,k} N_i(u)N_j(v)N'_k(w) \cdot c_{ijk_z} \end{pmatrix}$$

With the Gauss–Newton algorithm we iterate via the formula

$$J(Err(u,v,w)) \cdot \Delta \begin{pmatrix} u \\ v \\ w \end{pmatrix} = -Err(u,v,w)$$

and use Cramers rule for inverting the small Jacobian and solving this system of linear equations.

3.3 Parametrisierung sinnvoll?

- Nachteile von Parametrisierung
- Deformation ist um einen Kontrollpunkt viel direkter zu steuern.
- => DM–FFD?

4 | Scenarios for testing evolvability criteria using Freeform–Deformation (FFD)

4.1 Test Scenario: 1D Function Approximation

4.1.1 Optimierungszenario

- Ebene -> Template–Fit

4.1.2 Matching in 1D

- Trivial

4.1.3 Besonderheiten der Auswertung

- Analytische Lösung einzig beste
- Ergebnis auch bei Rauschen konstant?
- normierter 1–Vektor auf den Gradienten addieren

  – Kegel entsteht

4.2   Test Scenario: 3D Function Approximation

4.2.1   Optimierungsszenario

- Ball zu Mario

4.2.2   Matching in 3D

- alternierende Optimierung

4.2.3   Besonderheiten der Optimierung

- Analytische Lösung nur bis zur Optimierung der ersten Punkte gültig
- Kriterien trotzdem gut

5 | Evaluation of Scenarios

5.1 Spearman/Pearson–Metriken

- Was ist das?
- Wieso sollte uns das interessieren?
- Wieso reicht Monotonie?
- Haben wir das gezeigt?
- Statistik, Bilder, blah!

5.2 Results of 1D Function Approximation
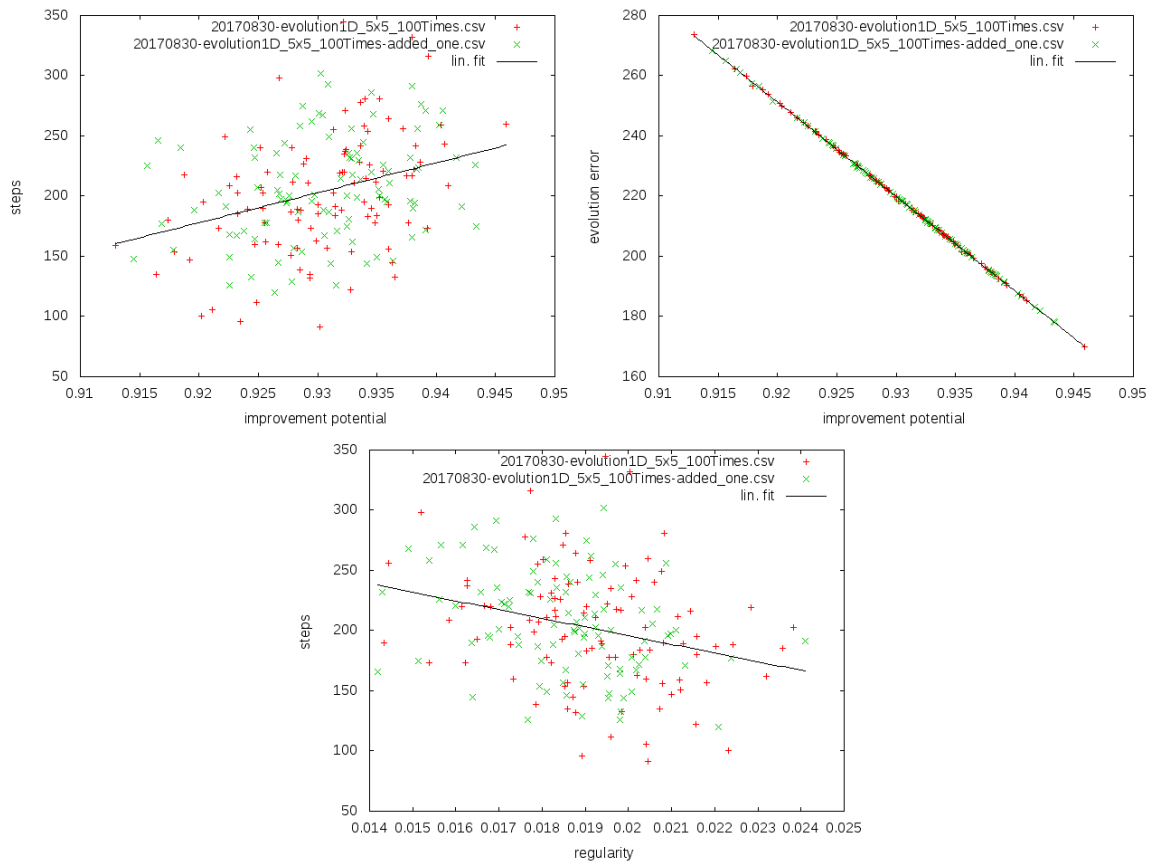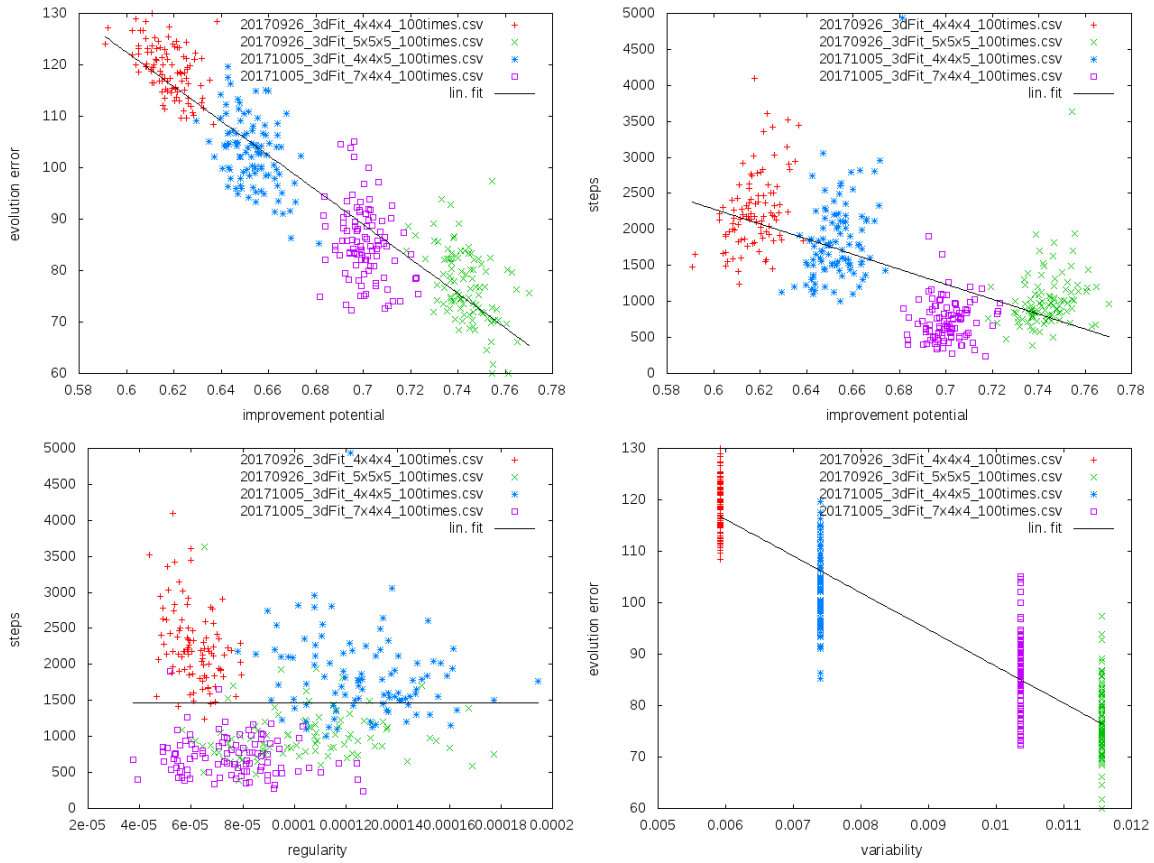
5.3 Results of 3D Function Approximation

Figure 5.1: Results 1D

Figure 5.2: Results 3D

6 | Schluss

HAHA .. als ob -.-

Appendix

A | Bibliography

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B | Abbreviations

FFD Freeform–Deformation

DM–FFD Direct Manipulation Freeform–Deformation

RBF Radial Basis Function

C | List of Figures

Todo list

Declaration of own work(?)

I hereby declare that this thesis is my own work and effort. Where other sources of information have been used, they have been acknowledged.

Improvement: write proper declaration..

Bielefeld, den October 9, 2017 ....

Stefan Dresselhaus